[7] S. B. Fonseca and A. J. Giarola, "Microstrip disk antennas, part 1: Efficiency of space wave launching," *IEEE Trans. Antennas Propagat.*, vol. AP-32, pp. 561–567, June 1984.
[8] J. S. Bagby, "Integral equation analysis of propagation in microstrip transmission lines," in *Dig. North American Radio Science Meeting* (Vancouver, Canada), June 1985, p. 178.
[9] A. Sommerfeld, *Partial Differential Equations in Physics* New York: Academic Press, 1964, pp. 236–265.
[10] R. W. P. King and C. W. Harrison, Jr., *Antennas and Waves: A Modern Approach*. Cambridge: M.I.T Press, 1969, pp. 15–17.
[11] J. R. Wait, *Electromagnetic Waves in Stratified Media*. New York: MacMillan, 1962, pp. 137–146.
[12] A. D. Yaghjian, "Electric dyadic Green's functions in the source region," *Proc. IEEE*, vol. 68, pp. 248–263, Feb. 1980.

Excitation of an Enclosed Lossy Cylinder by an Aperture Source

JAMES R. WAIT, FELLOW, IEEE

Abstract —**We present a formulation for an aperture-excited cylindrical structure of circular cross section and of finite length. The solution method is straightforward but it is sufficiently general to permit quantitative estimates of the internal heating. When the axial dependency of the fields is ignored, the solution reduces to one employed earlier in connection with hyperthermic heating of human torsos and limbs.**

I. Introduction

The controlled heating of conductive regions is an important objective in hyperthermia [1], not to mention related problems in designing microwave ovens [2]. It is our purpose to outline the solution for an idealized cylindrical model of the target. We will allow for the end effects of the cylinder in an explicit fashion. The source of the fields is an aperture of limited extent on the periphery of the cylinder. To simplify the analysis, the cylindrical target is taken to be homogeneous and to be characterized by a conductivity σ, a permittivity ϵ, and a permeability μ.

II. Formulation

As indicated in Fig. 1, cylindrical coordinates (ρ, ϕ, z) are employed with the cylinder bounded by $\rho = a$ and $z = \pm l$. For such a geometry, it is convenient to represent the electromagnetic fields within the cylinder in terms of z-directed electric and ma netic Hertz vectors [3]. Thus, for harmonic time dependence acc rding to $\exp(i\omega t)$,

$$E_\rho = \frac{\partial^2 U}{\partial\rho\,\partial z} - \frac{i\mu\omega}{\rho}\frac{\partial V}{\partial\phi} \qquad (1)$$

$$E_\phi = \frac{1}{\rho}\frac{\partial^2 U}{\partial\phi\,\partial z} + i\mu\omega\frac{\partial V}{\partial\rho} \qquad (2)$$

$$E_z = \left(-\gamma^2 + \frac{\partial^2}{\partial z^2}\right)U \qquad (3)$$

$$H_\rho = \frac{\partial^2 V}{\partial\rho\,\partial z} + \frac{\sigma + i\epsilon\omega}{\rho}\frac{\partial U}{\partial\phi} \qquad (4)$$

$$H_\phi = \frac{1}{\rho}\frac{\partial^2 V}{\partial\phi\,\partial z} - (\sigma + i\epsilon\omega)\frac{\partial U}{\partial\rho} \qquad (5)$$

$$H_z = \left(-\gamma^2 + \frac{\partial^2}{\partial z^2}\right)V \qquad (6)$$

Manuscript received July 12, 1986; revised September 29, 1986
The author is with the Electromagnetics Laboratory, ECE Department, University of Arizona, Tucson, AZ 85721.
IEEE Log Number 8611942.

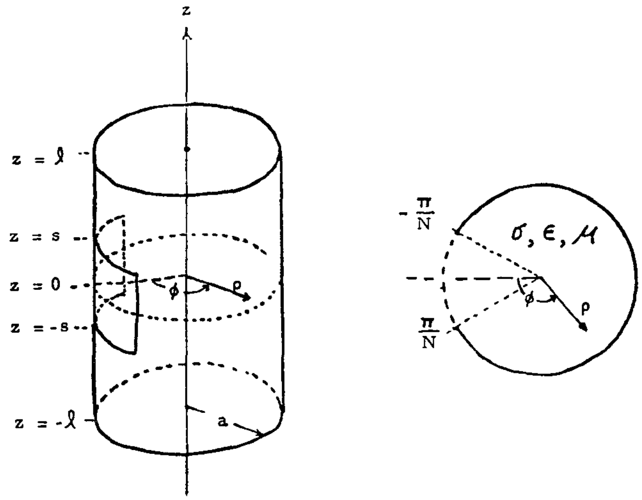

Fig. 1. Perspective and top view of cylindrical target of finite length showing aperture of angular width $2\pi/N$.

where U and V are scalar functions that satisfy

$$(\nabla^2 - \gamma^2)\,{}^U_V = 0 \qquad (7)$$

where

$$\gamma = [i\mu\omega(\sigma + i\epsilon\omega)]^{1/2} \qquad (8)$$

is the propagation constant defined such that $\operatorname{Re}\gamma > 0$.

To further simplify the discussion, we will specify that the aperture excitation at $\rho = a$, $0 < \phi < 2\pi$, and $-l < z < l$ is

$$E_z(a, \phi, z) = G(\phi, z) \qquad (9)$$

where $G(\phi, z)$ is an even function of both ϕ and z. Furthermore, we assume that the Fourier representation

$$G(\phi, z) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} g_{m,n}\cos m\phi\cos\frac{n\pi}{l}z \qquad (10)$$

is valid over the interval $0 < \phi < 2\pi$ and $-l < z < l$. Then, the Fourier coefficient is obtained from

$$g_{m,n} = \frac{\hat\epsilon_m\hat\epsilon_n}{\pi l}\int_0^l\int_0^\pi G(\phi, z)\cos m\phi\cos\frac{n\pi}{l}z\,d\phi\,dz \qquad (11)$$

where $\hat\epsilon_0 = 1$, $\hat\epsilon_m = 2$ $(m = 1, 2, 3 \cdots)$.

We also specify that, for $\rho = a$, $0 < \phi < 2\pi$, $-l < z < l$,

$$E_\phi(a, \phi, z) = 0$$

and, furthermore, that the bottom and top surfaces of the cylinder are perfectly conducting. That is, for $z = \pm l$, $0 < \rho < a$, and $0 < \phi < 2\pi$,

$$E_\rho(\rho, \phi, \pm l) = 0$$

and

$$E_\phi(\rho, \phi, \pm l) = 0.$$

(Later, we discuss the case where the ends of the cylinder are bounded by a perfect insulator.)

III. Field Representations

General solutions of U and V that are compatible with the present problem are found to be

$$U = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_m^n I_m(u_n\rho)\cos m\phi\cos\frac{n\pi}{l}z \qquad (12)$$

0018-9480/87/0200-0210$01.00 ©1987 IEEE

and

$$V = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} B_m^n I_m(u_n \rho) \sin m\phi \sin \frac{n\pi}{l} z \tag{13}$$

where $I_m(u_n \rho)$ is the modified Bessel function of the first type of order m and argument $u_n \rho$ [4]. Also, because (12) and (13) must satisfy (7), it follows that

$$u_n = \left[\gamma^2 + (n\pi/l)^2\right]^{1/2} \tag{14}$$

where we define Re $u_n > 0$. The coefficient A_m^n and B_m^n are yet to be determined.

Using (1), (2), and (3), we find that

$$E_\rho = -\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[\frac{n\pi}{l} u_n A_m^n I_m'(u_n \rho) + \frac{i\mu\omega}{\rho} \cdot m B_m I_m(u_n \rho)\right] \cdot \cos m\phi \sin \frac{n\pi}{l} z \tag{15}$$

$$E_\phi = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[\frac{m}{\rho} \cdot \frac{n\pi}{l} A_m^n I_m(u_n \rho) + i\mu\omega u_n B_m I_m'(u_n \rho)\right] \cdot \sin m\phi \sin \frac{n\pi}{l} z \tag{16}$$

and

$$E_z = -\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} u_n^2 A_m^n I_m(u_n \rho) \cos m\phi \cos \frac{n\pi}{l} z. \tag{17}$$

We are now in the position to invoke the aperture conditions at $\rho = a$. Thus

$$A_m^n \left[\frac{m}{a} \cdot \frac{n\pi}{l} \cdot I_m(u_n a)\right] + B_m^n [i\mu\omega u_n I_m'(u_n a)] = 0 \tag{18}$$

and

$$-A_m^n u_n^2 I_m(u_n a) = g_{m,n} \tag{19}$$

where $g_{m,n}$ is the double transform of the aperture field and is defined by (11). Clearly, A_m^n and B_m^n are now known once we specify the form of $G(\phi, z)$. We choose the active aperture to be rectangular in form of height $2s$ and angular width $2\pi/N$, where N is any integer greater than 1. In particular, we assume that

$$G(\phi, z) = E_0 \cos(\phi N/2) \quad \text{for} \begin{cases} -\pi/N < \phi < \pi/N \\ -s < z < s \end{cases}$$
$$= 0 \quad \text{for} \begin{cases} \pi/N < |\phi| < \pi \\ |z| > s \end{cases} \tag{20}$$

where E_0 is a constant. As indicated in Fig. 2, the electric field E_z vanishes at the edge of the aperture and has a maximum at the center line (i.e., $\phi = 0°$). On the other hand, E_z is assumed to be constant over the vertical extent of the aperture.

Equation (11) is now reduced to the form

$$g_{m,n} = \frac{\hat{\epsilon}_m \hat{\epsilon}_n}{\pi l} E_0 \int_0^s \int_0^{\pi/N} \cos\left(\frac{N}{2}\phi\right) \cos m\phi \cos \frac{n\pi}{l} z \, dz \, d\phi. \tag{21}$$

The integrations can be readily carried out to yield

$$g_{m,n} = \frac{\hat{\epsilon}_m \hat{\epsilon}_n}{n\pi^2} E_0 \sin\left(\frac{n\pi}{l} s\right) \times \begin{cases} \dfrac{1}{2} \dfrac{N}{(N/2)^2 - m^2} \cos \dfrac{m\pi}{N} \\ \text{or } \pi/(2N) \text{ if } m = N/2. \end{cases} \tag{22}$$

In this limiting two-dimensional situation, all variations in the z direction disappear and, as a consequence, the coefficient B_m^n vanishes in accordance with (18) for all values of m [5], [6].

The analysis given above is quite similar to that published by Ho, Guy, Sigelman, and Lehman [7] but they did not consider the conditions at the ends of their cylindrical model.

IV. Cylinder with Insulating Ends

The other case, of no less interest, is when the cylindrical target is truncated by a perfect insulator. That is, we assume that E_z within the cylinder vanishes at $z = \pm l$. In effect, we are saying that there is no current flow normally through the end surfaces. Clearly, this is an approximation which is valid if the displacement currents in the air are negligible compared with the total currents in the target. Such a condition will hold if $|\sigma + i\epsilon\omega| \gg \epsilon_0 \omega$, where ϵ_0 is the permittivity in the air.

For the conditions indicated, the solution is obtained in the same manner as when we have conducting end caps. Equations (1) through (8) are applicable, but now we want E_z to vanish at $z = \pm l$ for $0 < \rho < a$. Thus, in place of (9), we write

$$E_z(a, \phi, z) = F(\phi, z) \tag{24}$$

where

$$F(\phi, z) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} f_{m,n} \cos m\phi \cos\left(\frac{2n+1}{2l} \pi z\right) \tag{25}$$

where $f_{m,n}$ is the Fourier coefficient. In this case

$$f_{m,n} = \frac{2\hat{\epsilon}_m}{\pi l} \int_0^l \int_0^\pi F(\phi, z) \cos m\phi \cos\left(\frac{2n+1}{2l} \pi z\right) dz. \tag{26}$$

The general solutions of (7) are now taken to have the form

$$U = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} C_m^n I_m(u_n \rho) \cos m\phi \cos\left(\frac{2n+1}{2l} \pi z\right) \tag{27}$$

and

$$V = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} D_m^n I_m(u_n \rho) \sin m\phi \sin\left(\frac{2n+1}{2l} \pi z\right) \tag{28}$$

where now

$$u_n = \left[\gamma^2 + \left(\left(\frac{2n+1}{2l}\right) \pi\right)^2\right]^{1/2}.$$

The coefficients C_m^n and D_m^n are yet to be determined.

Using (1), (2), and (3) we find that

$$E_\rho = -\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[\left(\frac{2n+1}{2l}\right) \pi u_n C_m^n I_m'(u_n \rho) + \frac{i\mu\omega}{\rho} m D_m I_m(u_n \rho)\right] \cdot \cos m\phi \sin\left(\frac{2n+1}{2l} \pi z\right) \tag{30}$$

$$E_\phi = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[\frac{m}{\rho}\left(\frac{2n+1}{2l}\right) \pi C_m^n I_m(u_n \rho) + i\mu\omega u_n D_m I_m'(u_n \rho)\right] \cdot \sin m\phi \sin\left(\frac{2n+1}{2l} \pi z\right) \tag{31}$$

$$E_z = -\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} u_n^2 C_m^n I_m(u_n \rho) \cos m\phi \cos\left(\frac{2n+1}{2l} \pi z\right). \tag{32}$$

The aperture conditions $E_\phi(a, \phi, z) = 0$ and $E_z(a, \phi, z) = F(\phi, z)$ lead to the connecting relations

$$C_m^n \left[\frac{m}{a} \frac{2n+1}{2l} \pi I_m(u_n a)\right] + D_m^n [i\mu\omega u_n I_m'(u_n a)] = 0 \tag{33}$$

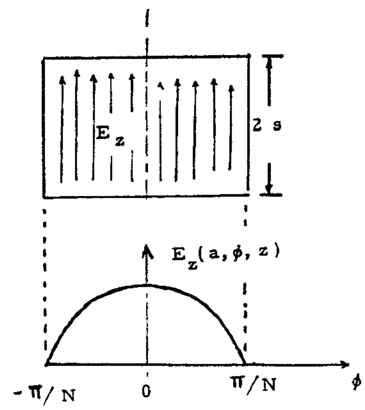

Fig. 2. Rectangular aperture and the vertical electric field distribution assumed in the analysis

and

$$-C_m^n u_n^2 I_m(u_n a) = f_{m,n}. \quad (34)$$

Again we choose a rectangular aperture with a cosinusoidal variation in the electric field such as given by the right-hand side of (20). But here, if the nonzero field E_z is constant within the aperture ($|z| < s$), we must insist that $l > s$; otherwise, (25) would be an improper representation.

Equation (26) is now reduced to

$$f_{m,n} = \frac{2\hat{\epsilon}_m}{\pi l} \int_0^l \int_0^\pi E_0 \cos\left(\frac{N}{2}\phi\right) \cos m\phi \cos\left(\frac{2n+1}{2l}\pi z\right) dz\, d\phi \quad (35)$$

which is evaluated to give

$$f_{m,n} = \frac{2\hat{\epsilon}_m E_0}{\left(n+\frac{1}{2}\right)\pi^2} \sin\left(\frac{2n+1}{2l}\pi s\right) \cdot \begin{cases} \frac{1}{2}\frac{N}{(N/2)^2 - m^2}\cos\frac{m\pi}{N} \\ \text{or } \frac{\pi}{2N} \text{ if } m = \frac{N}{2}. \end{cases} \quad (36)$$

This result is similar to (22). However, in the present case, we must keep $s < l$ to avoid a contradiction on the electric field at the ends of the cylinder. Of course, aperture conditions can be made to permit the E_z field at $\rho = a$ to vanish at $|z| = l$.

V. Concluding Remarks

The analytical results given here provide a means to calculate the internal fields in the cylinder for any assumed aperture distribution. The total power density P is then computed from

$$P = \sigma\left[|E_\rho|^2 + |E_\phi|^2 + |E_z|^2\right]/2 \quad (37)$$

in watts per meter^3. Of course, the results will depend on which model is used. If the cylinder is fully encased in a metal cast, except for the aperture, the first model considered above is indeed appropriate and the derived expressions for the fields are formally exact. The case where the metal only encloses the curved surfaces of the cylindrical target is really a much more complicated situation because the electromagnetic fields emanate to the external region. However, it is a good approximation to ignore these external fields insofar as making estimates of power deposition at points within the cylinder.

Finally, we should mention that the present analysis can be easily extended to any number of similar rectangular apertures arrayed around the periphery of the cylinder. As in the two-dimensional models [5], [6], we need merely to superimpose the results for a single aperture with due regard to angular location and individual excitation. Concentrically layered models pose no additional difficulty except for the increased complexity.

References

[1] J. C. Lin, Ed. Special issue on phased arrays for hyperthermia treatment of cancer, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 481–648, 1986.
[2] N. E. Bengtsson and T. Ohlsson, "Microwave heating in the food industry," *Proc. IEEE*, vol. 62, pp. 44–54, 1974.
[3] J. R. Wait, *Electromagnetic Radiation from Cylindrical Structures*. Oxford: Pergamon Press, 1959.
[4] J. R. Wait, *Electromagnetic Wave Theory*. New York: Harper and Row, 1986, ch. 3.
[5] J. R. Wait, "Focused heating in cylindrical targets," *IEEE Trans Microwave Theory Tech.*, vol. MTT-33, pp. 647–649, 1985.
[6] J. R. Wait and M. Lumori, "Focussed heating in cylindrical targets—Part II," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 357–359, 1986.
[7] H. S. Ho, A. W. Guy, R. A. Sigelmann, and J. F. Lehman, "Microwave heating of simulated human limbs by aperture sources," *IEEE Trans Microwave Theory Tech.*, vol MTT-19, pp. 224–231, 1971.

Noise Reduction in GaAs Schottky Barrier Mixer Diodes

K. M. KATTMANN, T. W. CROWE, MEMBER, IEEE, AND
R. J. MATTAUCH, FELLOW, IEEE

***Abstract* —The sensitivity of heterodyne receivers operating at millimeter and submillimeter wavelengths is limited by the noise produced in the mixer element. In this paper we investigate the presence of excess noise in GaAs Schottky barrier mixer diodes. Comparison of the measured noise data with that predicted from noise models indicates that these devices typically exhibit excess noise. An additional fabrication step, which removes several hundred angstroms from the GaAs surface before the anode contact is formed, greatly reduces this excess noise. This additional step is outlined, and experimental evidence is presented.**

I. Introduction

The sensitivity of heterodyne receivers operating at millimeter and submillimeter wavelengths is limited by the noise produced in the mixer element [1], [2], typically a GaAs Schottky barrier mixer diode. These devices are commonly cooled to cryogenic temperatures (of order 20 K) to minimize their contribution to the receiver noise. Thus, it is very important that all sources of excess noise in these devices be identified and eliminated. Schneider [3] has recently reported an anomalous peak occurring in the noise temperature versus forward current characteristic of small diameter (< 2 μm) Schottky barrier diodes. This excess noise has recently been linked to edge effects, possibly caused by stress at the $GaAs/SiO_2$ interface [4].

Manuscript received July 19, 1986; revised September 8, 1986. This work was supported in part by the National Science Foundation under Contract ECS-8412477.
The authors are with the Department of Electrical Engineering, Semiconductor Device Laboratory, University of Virginia, Thornton Hall, Charlottesville, VA 22901.
IEEE Log Number 8611628.